

Ministry of Science and Higher Education of the Russian Federation  
 Federal State Budgetary Educational Institution of Higher Education  
**Perm National Research Polytechnic University**

**APPROVED BY**

Pro-rector for Academic Affairs

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**ACADEMIC COURSE WORKING PROGRAM**

**Academic course:** Mathematics  
 (Name)

**Form of education:** Full-time  
 (Full-time /full-time – correspondence/correspondence)

**Level of higher education:** Bachelor's degree  
 (Bachelor's program/specialist program/  
 Master's program)

**Workload in hours (in credits):** 576 (16)  
 (Hours (CU))

**Training program (degree):** 21.03.01 Oil and Gas Engineering  
 (Code and denomination of degree)

**Direction:** Oil and Gas Engineering  
 (Title of curriculum)

## 1. GENERAL PROVISIONS

### 1.1. GOALS AND OBJECTIVES OF THE COURSE

The main goal is to form students' knowledge of mathematics aimed at solution of professional problems using methods of mathematical analysis, theory of differential equations, probability theory and math statistics, to assimilate, deepen and expand their mathematical knowledge, to develop a sustainable interest in the subject, as well as to form the ability of analytical thinking (GPC-1).

### 1.2. PRESCRIBED OBJECTS OF THE COURSE

Linear algebra. Matrices. Determinants. Systems of linear algebraic equations. Vector algebra. Vector quantities. Linear operations on vectors. Non-linear operations on vectors. Analytical geometry. Equation of a line on a plane. Equations of a plane, a line in space. Curves of the second order. Surfaces of the second order. Differential calculus. Introduction to mathematical analysis. Limit of a numerical sequence. Limit, function continuity. Derivative. Differential. Fundamental theorems of differential calculus. Function study. Theory of functions of several variables. Functions of several variables. Partial derivatives of functions of several variables. Extrema of a function of several variables. Integral calculus of a function of one variable. Indefinite integral. Classes of integrable functions. Definite integral. Geometric and physical applications of a definite integral. Differential Equations. Differential equations of the first order. Linear differential equations of higher orders. Systems of differential equations. Series. Number series. Alternating series. Functional series. Power series. Integral calculus of a function of several variables. Double integral. Triple integral. Contour integrals. Fourier series. Expansion of a function in a trigonometric Fourier series. Probability theory and mathematical statistics. The subject of probability theory. Methods for calculating probabilities. Random variables. Problems of mathematical statistics. Statistical estimates of distribution parameters.

### 1.3. STARTING CONDITIONS

Unstipulated

## 2. PLANNED RESULTS OF THE COURSE TRAINING

Competence	Indicator's Index	Planned Results of the Course Training (to know, to be able, to master)	Indicator of Attaining Competence which the planned results of training are correlated with	Means of Assessment
1	2	3	4	5
GPC-1	IA-1 <sub>gpc-1</sub>	<b>To know</b> the readiness to identify the natural scientific essence of the problems arising in the course of professional activity and the ability to attract the appropriate physical and mathematical apparatus to solve them	<b>Knows</b> basic concepts and provisions of the sections of higher mathematics which will be used in professional activities; basic concepts and methods of linear and vector algebra, analytical geometry; basic concepts and methods of differential and integral calculus; fundamentals of the theory of ordinary differential equations; series and their convergence, expansion of elementary functions in a series; harmonic analysis; basic concepts, methods and techniques of probability theory and mathematical statistics.	Exam
GPC-1	IA-3 <sub>gpc-1</sub>	<b>To master the skills</b> to present a scientific picture of the world adequate to the modern level of knowledge based on knowledge of the basic provisions, laws and methods of natural sciences and mathematics	<b>Masters the tools</b> for solving mathematical problems in their subject area; methods of mathematical analysis in solving professional problems; methods for solving algebraic and ordinary differential equations of the first and second order; methods of analytical geometry, linear algebra; methods of differential and integral calculus.	Practice work
GPC-1	IA-2 <sub>gpc-1</sub>	<b>To be able to</b> use the physical and mathematical apparatus for solving computational and analytical problems arising in the course of professional activity.	<b>Is able to</b> navigate in reference mathematical literature; use the basic laws of natural science disciplines, mathematical methods in professional	Test

1	2	3	4	5
			activity; to acquire new mathematical knowledge using modern educational and information technologies; solve typical problems in the main sections of the course, using the methods of higher mathematics; apply methods of differential calculus to solve extreme problems, study the behavior of functions and solve nonlinear equations; use the apparatus of linear algebra and analytical geometry; solve differential equations; calculate the probabilities of random events, compose and investigate the distribution functions of random variables, determine the numerical characteristics of random variables.	

### 3. FULL TIME AND FORMS OF ACADEMIC WORK

Form of academic work	Hours in all	Distribution in hours according to semesters		
		Number of semester		
		2	3	4
1. Holding classes (including results monitoring) in the form:				
1.1. Contact classwork, including:				
– lectures (L)	88	32	32	24
– laboratory work (LW)				
– practice, seminars and/or other seminar-type work (PW)	142	44	54	44
– control of self-work (CSW)	12	4	4	4
– test				
1.2. Students' self-work (SSW)	262	100	90	72
2. Interim/midterm assessment				
Exam	72	36		36
Grading test				
Test (Credit)				
Course Project (CP)				
Course Work (CW)				
<b>Workload in hours</b>	<b>576</b>	<b>216</b>	<b>180</b>	<b>180</b>

#### 4. COURSE CONTENTS

Course sections with brief contents	Full time of classroom activity in hours according to the forms			Full time of extracurricular work in hours according to the forms
	L	LW	PW	SSW
1	2	3	4	5
<b>Semester 2</b>				
I. Algebra and Geometry 1. Linear algebra. 2. Vector algebra. 3. Analytical geometry.	16		22	50
II. Introduction to math analysis 1. Differential calculus. Introduction to mathematical analysis.	16		22	50
<b>Total with regard to semester</b>	<b>32</b>		<b>44</b>	<b>100</b>
<b>Semester 3</b>				
I. Functions of several variables and integral calculus 1. Theory of functions of several variables. 2. Integral calculus of a function of one variable.	14		24	40
II. Differential equations and series 1. Differential equations. 2. Series.	18		30	50
<b>Total with regard to semester</b>	<b>32</b>		<b>54</b>	<b>90</b>
<b>Semester 4</b>				
I. Integral calculus of functions of several variables and harmonic analysis 1. Integral calculus of a function of several variables. 2. Fourier series.	12		22	36
II. Probability theory and math statistics 1. Probability theory and mathematical statistics.	12		22	36
<b>Total with regard to semester</b>	<b>24</b>		<b>44</b>	<b>72</b>
<b>Total with regard to the course</b>	<b>88</b>		<b>142</b>	<b>262</b>

#### Topics of exemplary practical work

Sl.No	Topic of practical work (seminars)
1	Determinants. Cramer's method for solving systems of linear algebraic equations.
2	Matrices. Matrix operations. Inverse matrix method for solving LAES.
3	Gaussian method for solving LAES.
4	Control work.
5	Vector. Vector properties. Linear operations on vectors. The problem of finding the coordinates of a vector in a new basis.
6	Scalar and vector product of two vectors. The geometric meaning of the vector product.
7	Triple product of three vectors. The geometric meaning of the triple product.
8	Equation of a line on a plane.
9	Plane. Equations of a line in space.
10	Curves of the second order. Surfaces of the second order.
11	Control work.

Sl.No	Topic of practical work (seminars)
12	Limit of a numerical sequence.
13	Function limit.
14	Function limit, function continuity.
15	Control work.
16	Derivative. The geometric and physical meaning of the derivative.
17	Derivative of a complex function.
18	Logarithmic differentiation. Implicit function differentiation. Differentiation of the parametric function.
19	Differential and its geometric meaning.
20	Derivatives of high order.
21	Function study using a derivative.
22	Control work.
23	Functions of several variables. Partial derivatives of functions of several variables.
24	Extrema of a function of several variables.
25	Directional derivative. Gradient.
26	Control work.
27	Indefinite integral. Replacement method in indefinite integral.
28	Method of integration by parts.
29	Integration of rational fractions.
30	Integration of some classes of trigonometric functions.
31	Integration of irrational functions.
32	Definite integral.
33	Geometric and physical applications of a definite integral.
34	Control work.
35	Elementary differential equations.
36	Homogeneous differential equations of the first order.
37	Linear differential equations of the first order.
38	Bernoulli's equation.
39	Exact differential equation.
40	Second-order differential equations admitting a reduction in order.
41	Linear homogeneous differential equations of the second order.
42	Linear inhomogeneous differential equations of the second order.
43	Systems of differential equations.
44	Control work.
45	Number series. Convergence criteria for numerical series
46	Alternating series.
47	Functional series. Power series.
48	Taylor and Maclaurain's series.
49	Control work.
50	Calculation of the double integral. Replacement method in double integral.
51	Double integral in polar coordinates.
52	Calculating surface area using double integral.
53	Triple integral.
54	Triple integral in cylindrical and spherical coordinate system.
55	Calculating body volume using triple integral.
56	Contour integrals.
57	Contour integrals.
58	Control work.
59	Examples of expansion of functions in Fourier series.

Sl.No	Topic of practical work (seminars)
60	Fourier series for a function with a period 2l.
61	Combinatorial elements.
62	Classical definition of probability.
63	Addition and multiplication theorems for probabilities. Dependent and independent events.
64	Total probability and Bayes formulas.
65	Bernoulli's formula.
66	Local and integral Laplace theorems.
67	Discrete random variable.
68	Continuous random variable.
69	Statistical estimates of distribution parameters.
70	Statistical testing of statistical hypotheses.
71	Control work.

Topics of exemplary laboratory practice

Sl. №	Topic of laboratory work

## 5. ORGANIZATIONAL AND PEDAGOGICAL CONDITIONS

### 5.1. EDUCATIONAL TECHNOLOGIES USED FOR COMPETENCES FORMATION

Holding lectures in the discipline is based on the active method of training in the process of which students are not passive but active participants of the lesson answering questions of the teacher. Teacher's questions are aimed at activating the process of learning material as well as at the development of logical thinking. The questions stimulating associative thinking and connecting new material with the previous one are formulated by the teacher in advance.

Practical lessons are held by realization of the method based on active training: problem areas are determined, groups are formed. The following aims are pursued in the process of practical education: use of definite disciplines knowledge and creative methods in solving problems and decision-making; students' skill-building of teamwork, interpersonal communication and development of leadership skills; consolidation of the basic theoretical knowledge.

### 5.2. STUDENTS' MANUAL FOR THE COURSE STUDY

Learning the course students are recommended to fulfill the following positions:

1. Learning of the discipline should be done systematically.
2. After learning one of the course unit with the help of the text-book or lecture notes it is recommended to reproduce the basic terms, definitions, notions of the unit from memory.

3. Special attention should be paid to the reports on practical studies, laboratory works and individual complex tasks for self-work.

4. The topics list for individual study is given by the teacher at the lectures. The teacher also provides students with literary sources (first of all, new ones in the periodical scientific literature) for a more detailed understanding of the issues presented at the lectures.

## 6. LIST OF TEACHING MATERIALS AND INFORMATION SUPPLY FOR STUDENTS' SELF WORK IN THE DISCIPLINE

### 6.1. PAPER-BASED COURSEWARE

Sl.№	Bibliographic entry (author, title, mode of publication, place, publishing house, year of publication, number of pages )	Number of copies in the library
<b>1. Basic literature</b>		
1	Mathematical Analysis / Ed. by J. M. Rassias. – Leipzig: B.G. Teubner Verl.-Ges., 1985.	1
2	Rudin Walter. Principles of Mathematical Analysis (International Series in Pure and Applied Mathematics) 3 <sup>rd</sup> ed. – NY: McGraw-Hill, 1976.	1
3	Mattuck Arthur. Introduction to Analysis. – Pearson, 1998.	1
4	Durrett Rick. Probability: Theory and Examples. 4 <sup>th</sup> ed. – Cambridge University Press, 2010.	1
5	Dudley R. M. Real Analysis and Probability. – Cambridge University Press, 2002.	1
<b>2. Additional literature</b>		
<b>2.1. Educational and scientific literature</b>		
1	Rosberg H.-J. Analytic Methods of Probability Theory / H.-J. Rosberg, B. Jesiak, G. Siegel. – Berlin: Akademie-Verl., 1985.	1
2	Marek I. Matrix Analysis for Applied Sciences : in two vol. / I. Marek, K. Zitny. – Leipzig: BSB Teubner, 1986.	1
3	Vol. 2. – Leipzig: , BSB Teubner, 1986. – (Matrix Analysis for Applied Sciences : in 2 vol.; Vol. 2).	1
4	Conway J. B. A Course in Functional Analysis. New York : Springer-Verlag, 1985. 406 p.	1
<b>2.2. Standardized and Technical literature</b>		
	No provision	
<b>3. Students' manual in mastering discipline</b>		
	No provision	
<b>4. Teaching and learning materials for students' self work</b>		
	No provision	



## 6.2. ELECTRONIC COURSEWARE

Kind of literature	Name of training tool	Reference to information resource	Accessibility of EBN (Internet/local net; authorized free access)
Book	Azbelev N.V., Maksimov V.P., Rakhmatullina L.F. Introduction to the Theory of Functional Differential Equations : Methods and Applications. New York : Hindawi Publ. Corr., 2007. 314 p.	<a href="https://elib.pstu.ru/vufind/Record/RUPSTUbooks126449">https://elib.pstu.ru/vufind/Record/RUPSTUbooks126449</a>	authorized free access

## 6.3. LICENSE AND FREE DISTRIBUTED SOFTWARE USED IN THE COURSE EDUCATIONAL PROCESS

Type of Software	Software branding
<i>Operating system</i>	Windows 10 (Azure Dev Tools for Teaching)
<i>Office application</i>	Adobe Acrobat Reader DC
<i>Image processing software</i>	Corel CorelDRAW Suite X4
<i>General purpose application software</i>	Mathematica Professional Version (license L3263-7820*)
<i>General purpose application software</i>	Microsoft Office Visio Professional 2016 (Azure Dev Tools for Teaching)
<i>General purpose application software</i>	WinRAR (license №879261.1493674)
<i>Management systems for projects, research, development, design, modeling and implementation</i>	Autodesk AutoCAD 2019 Education Multi-seat Stand-alone

## 6.4. MODERN PROFESSIONAL DATA BASES AND INQUIRY SYSTEMS USED IN THE COURSE EDUCATIONAL PROCESS

Branding	Reference to information resource
<i>Scopus database</i>	<a href="https://www.scopus.com/">https://www.scopus.com/</a>
<i>Web of Science Database</i>	<a href="https://www.webofscience.com/">https://www.webofscience.com/</a>
<i>Scientific electronic library database (eLIBRARY.RU)</i>	<a href="https://elibrary.ru/">https://elibrary.ru/</a>
<i>Scientific Library of the Perm National Research Polytechnic University</i>	<a href="https://lib.pstu/">https://lib.pstu/</a>
<i>Lan Electronic Library System</i>	<a href="https://e.lanbook.com/">https://e.lanbook.com/</a>
<i>Electronic library system IPRbooks</i>	<a href="https://www.iprbookshop.ru/">https://www.iprbookshop.ru/</a>
<i>Information resources of the Network ConsultantPlus</i>	<a href="https://www.consultant.ru/">https://www.consultant.ru/</a>
<i>Company database EBSCO</i>	<a href="https://www.ebsco.com/">https://www.ebsco.com/</a>

## 7. LOGISTICS OF THE COURSE EDUCATIONAL PROCESS

Type of classes	Name of the necessary basic equipment	Number of units
Lecture	Projector	1

## 8. FUND OF THE COURSE EVALUATING TOOLS

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Ministry of Science and Higher Education of the Russian Federation  
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## **FUND OF ESTIMATING TOOLS**

**For students' midterm assessment in the discipline  
"Mathematics"**

*Supplement to the Academic Course Working Program*

<b>Training program</b>	21.03.01 Oil and Gas Engineering
<b>Direction (specialization) of educational program</b>	Oil and Gas Engineering
<b>Graduate qualification</b>	Bachelor's degree
<b>Graduate academic chair</b>	Oil and Gas Technology
<b>Form of study</b>	Full-time studies
<b>Year:</b> 1, 2	<b>Semester:</b> 2, 3, 4

**Workload:**

in credits: 16 CU

in hours: 576

**The form of midterm assessment:**

**Exam** 2, 4 semester

**Test** 3 semester

**Fund of estimating tools** for midterm assessment of students' learning the subject "Mathematics" is the part (supplement) to the academic course working program. Fund of estimating tools for midterm assessment of students' learning the discipline has been developed in accordance with the general part of the fund of estimating tools for midterm assessment of the basic educational program which determines the system of the midterm assessment results and criteria of putting marks. Fund of estimating tools for midterm assessment of students' learning the subject determines the forms and procedures of monitoring results and midterm assessment of the subject leaning by the students.

### 1. LIST OF CONTROLLED RESULTS OF STUDYING DISCIPLINE, OBJECTS OF ASSESSMENT AND FORMS OF CONTROL

According to the Academic Course Working Program mastering course content is planned during three semesters (the second, third and fourth semester of curriculum) and is divided into eleven educational modules. Classroom activities, lectures as well as students' self-work are provided for every module. In the frames of mastering course content such competences as *to know, to be able, to master* pointed out in the ACWP are formed. These competences act as the controlled results of learning the discipline "Mathematics" (Table 1.1).

Monitoring of the acquired knowledge, abilities and skills is made in the frames of continuous assessment, progress check and formative assessment in the process of studying theoretical material, control works, reports on practical works, case-tasks and during examination. Types of control is given in Table 1.1

Table 1.1 – List of controlled results of learning the discipline

Controlled results of learning the discipline (KAS)	Type of control					
	Continuous assessment		Progress check		Formative assessment	
	D	AC	LWR/PWR	T/CW		Test
1	2	3	4	5	6	7
<b>Acquired knowledge</b>						
K.1 Basic definitions of linear algebra	D					TQ
K.2 Basic concepts of vector algebra	D					TQ
K.3 Basic definitions of analytical geometry: equations of a line on a plane, plane and a line in space	D					TQ
K.4 Definitions of the limit of a numerical sequence, function limit, derivative, differential. Fundamental theorems of differential calculus	D					TQ
K.5 Basic definitions of functions of several variables	D					TQ
K.6 Concepts of integral calculus	D					TQ
K.7 Concepts of theory of differential equations	D					TQ
K.8 Fundamentals of Series theory	D					TQ
K.9 Concepts of integral calculus of a function of several variables	D					TQ
K.10 Definition of trigonometric Fourier series	D					TQ

1	2	3	4	5	6	7
K.11 Basic definitions and theorems of probability theory and mathematical statistics	D	AC1				TQ
<b>Acquired abilities</b>						
A.1 Calculate determinants, perform algebraic operations on matrices, solve LAES using different methods			CT1	CW1		PT
A.2 Perform linear operations on vectors, find vector coordinates in a new basis, find scalar, vector and triple product of vectors				CW2		PT
A.3 Build equations of a line in a plane, a plane and equations of a line in space in various forms				CW3		PT
A.4 Calculate the limits of a numerical sequence, function limits, derivatives, differentials. Prove fundamental theorems of differential calculus. Study a function using derivative theory.			PWR1	CW4		PT
A.5 Calculate partial derivatives of a function. Find an extrema of a function of several variables.			CT2			PT
A.6 Calculate integrals of functions of one variable.			CT3	CW5		PT
A.7 Solve differential equations of the first and higher orders, systems of differential equations				CW6		PT
A.8 Investigate the question of convergence of a numerical and alternating series. Expand the function in a series and solve the problem of the convergence of the functional series.				CW7		PT
A.9 Calculate double and triple integrals.			PWR2			PT
A.10 Expand the function in a trigonometric Fourier series.						
A.11 Solve the problems of probability theory and mathematical statistics				CW8		PT
<b>Mastered skills</b>						
S.1 Basic methods of linear algebra			CT1	CW1		
S.2 Basic techniques of working with vectors				CW2		
S.3 Methods of building of a line in a plane, a plane and equations of a line in space				CW3		
S.4 The main methods for finding solutions to limits and derivatives			PWR1	CW4		
S.5 Basic principles and methods of functions of several variables			CT2			
S.6 Basic methods of integral calculus			CT3	CW5		
S.7 Basic methods of theory of differential equations				CW6		
S.8 Basic techniques of working with numerical and functional series				CW7		
S.9 Basic methods of integral calculus of a function of several variables			PWR2			
S.10 Method of expansion of a function in a trigonometric Fourier series.						
S.11 Basic methods and principles of probability theory and mathematical statistics				CW8		

*D – topic discussion; AC – colloquium (discussion of theoretical material, academic conference); CT – case-task (individual task); LWR – report on laboratory work; PWR – report on practical work; T/CW – progress check (control work); TQ – theoretical question; PT – practical task; CT – complex task of grading test.*

Final assessment of the learned discipline results is the midterm assessment which is made in the form of test taking into consideration the results of the running and progress check.

## **2. TYPES OF CONTROL, STANDARD CONTROL TASKS AND SCALES OF LEARNING RESULTS ASSESSMENT**

Continuous assessment of the academic performance is aimed at maximum effectiveness of educational process, at monitoring students' specified competencies formation process, at increase of learning motivation and provides the assessment of mastering the discipline. In accordance with the regulations concerning the continuous assessment of the academic performance and midterm assessment of students taught by the educational programs of Higher education – programs of the Bachelor's Course, Specialists' and Master's Course the next types of students' academic performance continuous assessment and its periodicity is stipulated in PNRPU:

- acceptance test, check of the student's original preparedness and his correspondence with the demands for the given discipline learning;

- continuous assessment of mastering the material (the level of mastering the component "to know" defined by the competence) at every group studies and monitoring of lectures attendance;

- interim and progress check of students' mastering the components "to know" and "to be able" of the defined competences by computer-based or written testing, control discussions, control works (individual home tasks), reports on laboratory works, reviews, essays, etc.

Discipline progress check is conducted on the next week after learning the discipline module, while the interim control is made at every monitoring during the discipline module study;

- interim assessment, summarizing of the current students' performance at least once a semester in all disciplines for every training program (specialty), course, group;

- control of retained knowledge.

### **2.1. CONTINUOUS ASSESSMENT OF EDUCATION**

Continuous assessment of learning is made in the form of discussion or selective recitation on every topic. According to the four-point system the results of assessment are put into the teachers' note-book and are considered in the form of integral mark in the process of the midterm assessment.

### **2.2. PROGRESS CHECK**

For the complex assessment of the acquired knowledge, abilities and skills (Table 1.1) progress check is carried out in the form of case-tasks (individual

tasks), reports on practical work, colloquium and midterm control works (after learning a certain section of the discipline).

### 2.2.1. Case-tasks (individual tasks)

It is planned 3 case-tasks all in all. The first CT is realized with respect to the module 1 "Linear algebra", the second CT – with respect to the module 5 "Theory of functions of several variables" and the third CT – with respect to the module 6 "Integral calculus of a function of one variable".

#### Standard tasks of the first CT:

1. Calculate the determinant in two ways: a) decomposition in terms of row or column elements, b) reducing to zero the elements of a row or column

$$1) \begin{vmatrix} d & -a & c \\ -2d & 3a & -2c \\ -b & c & b \end{vmatrix}; \quad 2) \begin{vmatrix} a & d & b & c \\ a+c & d & d+b & c \\ b & -c & d & b \\ 2b & -c & d-b & b \end{vmatrix};$$

$$3) \begin{vmatrix} a & c & b & a \\ c & -d & b & d \\ 2a-b & 2c & 2b & 2a+c \\ -2c & d & -b & -2d \end{vmatrix}.$$

2. Find the matrices product

$$1) \begin{pmatrix} a & b & c \\ -a & b & -c \\ a & d & d \end{pmatrix} \cdot \begin{pmatrix} c & d & b \\ -c & d & -b \\ c & b & -c \end{pmatrix}; \quad 2) \begin{pmatrix} b & a & a \\ b & c & -c \\ a & b & c \end{pmatrix} \cdot \begin{pmatrix} b & c & a \\ -b & a & -a \\ b & d & -c \end{pmatrix}$$

3. Find the inverse of the given matrix. Make a check.

$$A = \begin{pmatrix} a & b & c \\ a & b & d \\ d & c & b \end{pmatrix}$$

4. Solve a system of linear algebraic equations using: 1) Cramer's formulas, 2) the matrix method, 3) the Gauss method.

$$\begin{pmatrix} d & b & c \\ a & c & b \\ b & d & a \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} e \\ f \\ g \end{pmatrix}.$$

**5. Solve a homogeneous system of linear equations**

$$\begin{pmatrix} a & b & c \\ b & b & d \\ c & d & a \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

**6. Solve a system of linear algebraic equations by the Gauss method. Make a check.**

$$\begin{pmatrix} -b & c-a & b-a & d \\ b & a & d & b \\ a & b & c & a \\ b & c & b & d \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} e-g \\ g+bd \\ f+ad \\ e+bd \end{pmatrix}.$$

The values of the numbers  $a, b, c, d, e, f, g$  for each variant are given in the following table:

Variant	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	1	-1	3	-3	4	-4	5	-5	1	-1	1	-1	3	-3	4
b	1	2	1	-2	1	3	-1	1	2	3	1	2	1	-2	1
c	2	1	-2	1	3	1	3	-3	3	2	2	1	-2	1	-3
d	1	-1	1	-1	1	-1	1	1	1	-1	-2	2	2	-2	2
e	6	6	8	8	14	14	15	5	14	14	3	3	11	11	18
f	5	5	5	5	22	22	19	19	13	13	5	5	5	5	22
g	4	-5	-2	5	17	-19	9	11	7	-8	1	1	-1	7	18

Variant	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
a	-4	5	-5	1	-1	-1	-1	3	-3	4	-4	5	-5	1	-1
b	3	-1	1	2	3	1	2	1	-2	1	3	-1	1	2	3
c	1	3	-3	3	2	2	1	-2	1	3	1	3	-3	3	2
d	-2	2	2	-2	-2	3	-3	3	-3	-3	-3	4	4	4	-3
e	18	20	0	11	15	8	8	14	14	-2	22	30	-10	17	16
f	22	19	19	13	13	5	5	5	5	22	22	19	19	13	13
g	-22	8	12	1	-11	6	-9	0	9	13	-25	6	14	13	-14

**Standard tasks of the second CT:**

**Variant 1**

1. Find and draw the range of the function:  $u = \sqrt{x^2 + y^2 - 2x - 3}$ .
2. Find partial derivatives of the first and second order, total differentials  $dz$  and  $d^2z$  of the function  $z = ye^{xy}$ . Find the values  $d^2z$  at the point  $M_0(0; 1)$ .
3. Find the derivative  $\frac{\partial^2 z}{\partial x \partial y}$  of the function  $z = \cos(e^{2y} - 2x)$ .
4. Test the function for extremum  $z = 3x^2 - x^3 + 3y^2 + 4y$ .

**Standard tasks of the third CT:**

- Find the indefinite integral (**tasks 1–21**)
- Calculate the definite integral (**tasks 22, 23**)
- Investigate the convergence of the improper integral (**tasks 24, 25**)

№ 1

$$\begin{aligned}
 &1) \int e^{4\sin x - 3} \cos x \, dx; \quad 2) \int e^{7+8x^3} x^2 \, dx; \quad 3) \int \frac{(4-7x)dx}{(4x-3.5x^2)^2}; \quad 4) \int \frac{5-7\ln^2 x}{2x} dx; \\
 &5) \int x \cos(3-x^2) dx; \quad 6) \int \frac{2x dx}{\sqrt{x^4-3}}; \quad 7) \int (4-3x)e^{-3x} dx; \quad 8) \int (x^2+6)\cos 2x dx; \\
 &9) \int e^{2x} \sin 3x dx; \quad 10) \int \frac{x^4+2x^3-x^2-6x-2}{x(x-1)(x-2)} dx; \quad 11) \int \frac{x^3+6x^2+13x+9}{(x+1)(x+2)^3} dx; \\
 &12) \int \frac{x^3+4x^2+4x+2}{(x+1)^2(x^2+x+1)} dx; \quad 13) \int \frac{dx}{\sin^2 x (1+\cos x)}; \quad 14) \int \frac{\sin 2x}{\sqrt{\cos^2 x - 1}} dx; \\
 &15) \int \sin \frac{8x}{3} \cos \frac{7x}{3} dx; \quad 16) \int \sin^4 \frac{x}{2} dx; \quad 17) \int \frac{\cos^3 x}{\sqrt[5]{\sin^3 x}} dx; \quad 18) \int \frac{dx}{\sqrt{\sin x \cos^7 x}}; \\
 &19) \int \frac{dx}{\sqrt[3]{(2x+1)^2} - \sqrt[3]{2x+1}}; \quad 20) \int \frac{(5x+6)dx}{\sqrt{x^2+4x+5}}; \quad 21) \int x^{\frac{7}{4}} \sqrt{1+\sqrt{x}} dx; \\
 &22) \int_0^{16} \sqrt{256-x^2} dx; \quad 23) \int_{-3}^{-2} (2x+5) \sin \frac{2\pi x}{3} dx; \quad 24) \int_0^{\infty} x e^{-2x^2} dx; \quad 25) \int_{-\pi/4}^{3\pi/4} \frac{dx}{\cos^2 x}.
 \end{aligned}$$

**2.2.2. Reports on practical work**

It is planned 2 practicals all in all. Standard topics of practical work are: “Function study and plotting using the theory of derivatives” and “Double, triple and contour integrals”

Presentation of practical work is made by the student individually. Standard scale and criteria of assessment are given in the general part of FET of the educational program.

**Standard tasks of the first PWR:****Variant 1**

1. Study of these functions using differential calculus methods and build the plots:

$$a) y = \frac{x}{\sqrt[3]{x^2-1}}$$

$$b) y = e^{x^2-2x}$$



b)  $y = \sin x + \frac{1}{3} \sin 3x$

2. Find the maximum and minimum of a function in a given interval:

$$y = \frac{3}{1+x^2}, x \in [-1; 2]$$

**Standard tasks of the second PWR:**

**Variant 1**

1. Change the order of integration  $\int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^0 f(x, y) dx + \int_{-1}^0 dy \int_{-\sqrt{-y}}^0 f(x, y) dx$ .

2. Calculate the double integral  $\iint_D xy dx dy$  over the region D determined by

the following conditions  $\begin{cases} xy = 1 \\ x + y = \frac{5}{2} \end{cases}$ .

3. Calculate the area of the region D bounded by the curve using the double integral  $(x^2 + y^2)^3 = a^2(x^4 + y^4)$ .

4. Calculate the volume of the body V bounded by the surfaces using the double integral  $\begin{cases} x^2 + y^2 + z^2 = 2z \\ x^2 + y^2 = z^2 \end{cases}$ . The density of the body V is considered equal to one.

5. Calculate the triple integral  $\iiint_V \frac{dx dy dz}{(1+x+y+z)^3}$  over the spatial

domain V bounded by surfaces  $\begin{cases} x+z=3 \\ y=2 \\ x=0, y=0, z=0 \end{cases}$ .

6. Calculate the integral  $\iiint_V (x^2 + y^2) dx dy dz$ , if the region V is defined by the inequalities  $z \geq 0, r^2 \leq x^2 + y^2 + z^2 \leq R^2$ .

7. Calculate the mass of a body bounded by a surface  $T: \begin{cases} 64(x^2 + y^2) = z^2, x^2 + y^2 = 4 \\ y=0, z=0, (y \geq 0, z \geq 0) \end{cases}$  and having density  $\mu = \frac{5}{4}(x^2 + y^2)$ .

8. Calculate the contour integral of the first kind  $\int_L \frac{x}{y} d\ell$ , if L is the arc of circle  $x = 2 \sin t, y = 2 \cos t, \frac{\pi}{6} \leq t \leq \frac{\pi}{3}$ .

9. Find the center of gravity of one arch of the cycloid  $x = 2(t - \sin t)$ ,  $y = 2(1 - \cos t)$ ,  $0 \leq t \leq 2\pi$  assuming density equal to one.

10. Calculate the contour integral of the second kind  $\int_L (6 - y)dx + xdy$ , where  $L$  is the arc of cycloid  $x = 3(t - \sin t)$ ,  $y = 3(1 - \cos t)$ ,  $0 \leq t \leq 2\pi$ .

11. Calculate using Green's formula the contour integral  $\int_L x^2 y dx - xy^2 dy$  over the circle  $L$  centered at the origin of the radius  $R$ , with the positive direction of the traversal.

12. Calculate the surface integral of the first kind  $\iint_S xyz ds$  over the spatial region  $S = \{(x, y, z)\}$ , determined by the conditions  $\left\{ \begin{array}{l} x + y + z = 1, \\ x \geq 0, y \geq 0, z \geq 0 \end{array} \right\}$ .

13. Calculate the contour integral by Stokes' formula  $\oint_L y dx + z^2 dy + x^2 dz$  where  $L$  is the circle along which the plane  $z = \sqrt{3}$  intersects the sphere given by the equation  $x^2 + y^2 + z^2 = 4$ .

### 2.2.3. Colloquium

1 colloquium on "Probability theory" is planned. Questions to the colloquium:

1. Combinatorial elements (placement, permutation, combinations). Examples.
2. Combinatorial rules. Examples.
3. Events, types of events, operations on events.
4. The classical definition of the probability of an event. Probability properties.
5. Joint and incompatible events. Probability addition theorems.
6. Conditional probability. Probability multiplication theorems for dependent and independent events.
7. Formula of total probability. Bayes' formula.
8. Bernoulli's formula. Examples.
9. Bernoulli asymptotic formulas.
10. Random variables. Distribution laws of random variables.
11. Distribution functions of discrete random variable. Numerical characteristics of DRV.
12. Distribution laws of DRV.
13. Continuous random variable. Distribution function and distribution density of CRV.

14. Numerical characteristics of CRV.
15. Distribution laws of CRV.

#### **2.2.4. Midterm control work**

According to ACWP 8 midterm control works (CW) is planned to be realized after learning the educational modules of the discipline by the students.

The first CW is realized with respect to the module 1 “Linear algebra”, the second CW – with respect to the module 2 “Vector algebra”, the third CW – with respect to the module 3 “Analytical geometry”, the fourth CW – with respect to the module 4 “Differential calculus. Introduction to mathematical analysis”, the fifth CW – with respect to the module 6 “Integral calculus of a function of one variable”, the sixth CW – with respect to the module 7 “Differential Equations”, the seventh CW – with respect to the module 8 “Series” and the eighth CW – with respect to the module 11 “Probability theory and mathematical statistics”.

#### **Standard tasks of the first CW:**

##### Variant 1

1. Perform the following operations

$$\begin{pmatrix} 5 & 3 & -1 \\ -1 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & -2 \\ 7 & 2 \\ 1 & 3 \end{pmatrix} - 4 \cdot \begin{pmatrix} 2 & 3 \\ -6 & 1 \end{pmatrix}$$

2. Solve the system in three ways (Gauss method, Cramer's rule, matrix method)

$$\begin{cases} 3x - 2y + z = 3; \\ 4x - 3y + 4z = 9; \\ x - 2y - 3z = -7. \end{cases}$$

#### **Standard tasks of the second CW:**

##### Variant 2.

1. The following points are given:  $A(-3;1;4)$ ;  $B(2;2;1)$ ;  $C(3;-1;1)$ ;  $D(1;-2;5)$ .

Find:

$$\begin{aligned} \text{a) } & |3\overline{AD} + \overline{BC}|; & \text{б) } & (\overline{DB}, 3\overline{CA} - \overline{CB}); & \text{в) } & \text{пр}_{\overline{CA}}(\overline{DB} - \overline{AC}); \\ \text{г) } & \cos\left(\widehat{\overline{BC}, \overline{DA}}\right); & \text{д) } & [(3\overline{CA} - 2\overline{AB}) \times \overline{AD}] & \text{е) } & S_{BCD}; & \text{ж) } & H_A \end{aligned}$$

2. Find the value of  $k$ , if vectors  $\overline{p} = \{-2; k; 3\}$  и  $\overline{q} = \{4; -5; 3k + 1\}$  are perpendicular.

3. Find the value of  $m$ , if vectors  $\overline{p} = m\overline{i} + 2\overline{j}$  и  $\overline{q} = 2m\overline{j} + 4\overline{i}$  are collinear.

4. Find the value of  $\alpha$ , if vectors  $\overline{p} = \{-1; \alpha; 1\}$ ;  $\overline{q} = \{2; 1; -2\}$ ;  $\overline{s} = \{4; 1; -3\}$  are coplanar.

**Standard tasks of the third CW:**Variant 3.

- I. 1. Write down the equation of the plane passing through the point  $A(-3;1;-2)$  parallel to the vectors  $\bar{a} = \{-4;5;3\}$  and  $\bar{b} = \{-2;7;9\}$ .
2. Write down the equation of the straight line passing through the point  $C(-1; 6; 3)$  parallel to the vector  $\bar{b} = \{-2;7;9\}$ .
3. Find the distance from a point on a line  $B(3;1;-1)$  to a plane  $6x - 2y + 7z - 4 = 0$ .
4. Write down the equation of the plane passing through the point  $C(-1;6;3)$  perpendicular to the line  $\frac{x+5}{-2} = \frac{y-3}{4} = \frac{z+3}{1}$ .
5. Write down the canonical equation of the line  $\begin{cases} 3x - 5y + 2z - 4 = 0; \\ 7x - y + 4z = 0. \end{cases}$
- II. 1. Write down the equation of a line passing through a point  $E(-2;3)$  parallel to a line  $7x - 5y + 9 = 0$ .
2. Write down the equation of the line passing through the point  $T(4;-3)$  perpendicular to the line  $\frac{x+2}{5} = \frac{y-1}{-2}$ .
3. Write down the equation of a line passing through point  $B(-1;1)$  and forming an angle  $45^\circ$  in with a line  $4x - 2y + 3 = 0$ .

**Standard tasks of the fourth CW:**Variant 3.

1.  $\lim_{n \rightarrow \infty} \frac{4n - 3n^7 + 2n^9 - 19}{13 + 17n^5 - 19n^{10} + 18n^3}$ ;
2.  $\lim_{n \rightarrow \infty} \frac{\sqrt[5]{12n^{16} - 8n^{25} + 15} - \sqrt[7]{n^4 - 14n^2 + 5n^{11} + 6}}{\sqrt[3]{17n^{15} - 19n^{10} + 4}}$ ;
3.  $\lim_{n \rightarrow \infty} (\sqrt{4n^2 - 5n + 6} - \sqrt{4n^2 + 3n - 9})$
4.  $\lim_{n \rightarrow \infty} \left( \frac{5n^2 + 12n - 4}{5n^2 + 5n + 2} \right)^{3n+1}$
5.  $\lim_{n \rightarrow \infty} \left( \frac{3n - 14}{12n + 6} \right)^{17n-9}$
6.  $\lim_{n \rightarrow \infty} \left( \frac{3n - 4}{12n + 6} \right)^{17-n}$
7.  $\lim_{n \rightarrow \infty} \left( \frac{19n - 11}{4n + 5} \right)^{\frac{2n+4}{6n-3}}$
8.  $\lim_{x \rightarrow -1} \frac{3x^2 + 7x + 4}{7 + x - 5x^2 + x^3}$

$$9. \lim_{x \rightarrow -5} \frac{\sqrt{6+x} - 1}{7 - \sqrt{24+x^2}}$$

$$11. \lim_{x \rightarrow 0} \frac{\ln(1-3x^2)}{\operatorname{tg} 5x \cdot (e^{4x} - 1)}$$

$$10. \lim_{x \rightarrow -5} (6+x)^{\frac{3}{5+x}}$$

$$12. \lim_{x \rightarrow \frac{5\pi}{2}} \frac{1 - \sin x}{\operatorname{ctg} 7x \cdot (e^{2x} - e^{5\pi})}$$

### Standard tasks of the fifth CW:

#### Variant 4

Solve the following indefinite integrals:

$$1) \int 2^{-\cos x} \sin x dx; \quad 2) \int \frac{7-2x}{x^2-7x} dx; \quad 3) \int (3 + \ln^2 x) \frac{dx}{x};$$

$$4) \int x \sin(5x^2 - 1) dx; \quad 5) \int x^2 e^{7-2x^3} dx; \quad 6) \int \frac{x dx}{\sqrt[3]{5+2x^2}};$$

$$7) \int (4x-2) \cos 2x dx; \quad 8) \int (x^2+1) e^{3x} dx;$$

$$9) \int e^{2x} \cos 4x dx; \quad 10) \int \frac{2x^3+5}{x^2-x-2} dx; \quad 11) \int \frac{\cos x dx}{(1-\cos x)^3};$$

$$12) \int \frac{1+\operatorname{tg}^2 x}{(\operatorname{tg} x - 1)^3} dx; \quad 13) \int \sin \frac{5x}{2} \sin \frac{3x}{2} dx;$$

$$14) \int \sin^2 \frac{x}{2} \cos^4 \frac{x}{2} dx; \quad 15) \int \frac{\sin^5 x}{\sqrt{\cos x}} dx; \quad 16) \int \frac{x dx}{\sqrt{4-6x-3x^2}}.$$

### Standard tasks of the sixth CW:

#### 5 Variant

1. Find the general integral of a differential equation

$$x\sqrt{1+y^2} + yy'\sqrt{1+x^2} = 0.$$

2. Find the general integral of a differential equation.

$$y' = \frac{x+2y}{2x-y}.$$

3. Find a solution of the following Cauchy problem

$$y' - \frac{y}{x} = x \sin x, \quad y\left(\frac{\pi}{2}\right) = 1.$$

4. Find the general integral of a differential equation

$$xy' = 4\sqrt{2x^2 + y^2} + y.$$

**Standard tasks of the seventh CW:****5 Variant**

1. Study the series convergence  $\sum_{n=1}^{\infty} \frac{n}{7n+3}$ ;

2. Study the series convergence  $\sum_{n=1}^{\infty} \frac{n \cdot (-1)^{n-1}}{\sqrt{n+1}}$ ;

3. Find the convergence interval and convergence radius of power series  $\sum_{n=1}^{\infty} \frac{n^2 x^n}{7^n \cdot (n+1)}$ ;

4. Decompose the function  $f(x) = \ln(1+2x)$  into Taylor's series according to powers  $(x-3)$ . Find the convergence domain of the obtained series.

**Standard tasks of the eighth CW:****1 Variant**

1. The bag contains 5 identical cubes. One of the following letters is written on all faces of each cube: o, n, p, s, t. Find the probability that the word "sport" can be read on the cubes taken out one at a time and located "in one line".

2. In a batch of 10 parts 8 are standard. Find the probability where among the randomly extracted 2 parts at least one is standard.

3. There are radio tubes in two boxes. The first box contains 12 tubes, of which 1 is non-standard; in the second 10 lamps, of which 1 is non-standard. A lamp was taken at random from the first box and put into the second. Find the probability where the lamp taken out of the second box at random will be non-standard.

4. There are 6 motors in the workshop. For each motor, the probability that it is currently on is 0.8. Find the probability where 4 motors are currently on.

5. The probability of hitting a target by a shooter with one shot is 0.75. Find the probability where at 100 shots the target will be hit at least 70 and no more than 80 times.

Standard scale and criteria of the results of the midterm control work assessment are given in the general part of FET of the educational program.

### 2.3. FULFILLMENT OF THE COMPLEX INDIVIDUAL SELF-WORK TASK

Individual complex task for the students is used for assessment their skills and abilities acquired in the process of learning the discipline in which the course project or course paper is not stipulated.

Standard scale and criteria of assessment of the individual complex task presentation are given in the general part of FET of the educational program.

## 2.4. MIDTERM ASSESSMENT (FINAL CONTROL)

Admission for midterm assessment is made according to the results of continuous assessment and progress check. Preconditions for admittance are successful presentation of all case-tasks (individual tasks), reports on practical work, colloquium and midterm control works and positive integral estimation with respect to the results of continuous assessment and progress check.

### *2.4.1. Midterm assessment procedure without additional evaluation testing*

Midterm assessment is made in the form of test. Credit on the discipline is based on the results of the previously fulfilled by the student individual tasks on the given discipline.

Criteria of putting the final mark for the components of competences in the process of midterm assessment made in the form of test are given in the general part of FET of the educational program.

### *2.4.2. Midterm assessment procedure followed by evaluation testing*

In definite cases (for example, in case of re-attestation of the discipline) midterm assessment in the form of the test on this discipline can be made as the card-based evaluation test. Every exam card includes theoretical questions (TQ) aimed at control of the acquired knowledge, practical tasks (PT) aimed at mastered abilities, and complex tasks (CT) aimed at control of the acquired skills of all declared competences.

The exam card is formed so that the included questions and practical tasks could estimate the level of maturity of **all** declared competences.

#### **2.4.2.1. Standard questions and tasks the discipline testing**

##### **Standard questions for the acquired knowledge control:**

1. Functions of several variables. The range of definition of functions of several variables. Partial derivatives of a function of two variables.
2. Partial derivatives of a function of three variables. Examples.
3. Directional derivative and gradient of functions of several variables.
4. Research of functions of several variables for extremum.
5. Ordinary differential equations with separable variables.
6. Homogeneous differential equations of the 1st order.
7. Linear inhomogeneous DE of the 1st order.
8. Bernoulli's equations.
9. ODE admitting reduction of order.

10. Homogeneous ODE of the 2nd order with constant coefficients.
11. Inhomogeneous ODE of the 2nd order with constant coefficients.
12. Improper integral.
13. The definite integral and its applications.
14. Indefinite integral. Table of Indefinite integrals.
15. The replacement method in an indefinite integral.
16. Formula of integration by parts in the indefinite integral.
17. Methods for solving indefinite integrals from functions containing trigonometric functions.
18. Integration of fractions of a special kind.
19. Integration of fractional-rational functions.
20. Integration of irrational functions.

**Standard questions and practical tasks for the mastered abilities control:**

Examination paper № 2

1. Determine the type of differential equation and find its general integral:

$$y - xy' = 1 + x^2 y'.$$

2. Research the function for extremum:

$$z = xy(6 - x - y).$$

**Standard complex tasks for the acquired skills control:**

Examination paper № 5

1. Homogeneous differential equations of the 1st order.
2. Research the function for extremum:  $z = x^3 + y^3 - 15xy$ .
3. Find the area of a plane figure bounded by curves:  $y = x^2 - 1$ ,  $y = x + 1$ .

**2.4.2.2. Scales of test assessment of educational achievements**

Evaluation of discipline achievements in the form of maturity level of the components *to know*, *to be able*, *to master* of the declared competences is made according to the four-point assessment scale.

Standard scale and criteria of estimating educational achievements in the process of testing for the components *to know*, *to be able*, *to master* are given in the general part of FET of educational program.



### **3. ASSESSMENT CRITERIA FOR COMPONENTS AND COMPETENCES LEVEL OF MATURITY**

#### **3.1. ASSESSMENT OF COMPETENCES COMPONENTS LEVEL OF MATURITY**

While estimating the level of competences maturity by selective control in the process of testing it is considered that *the mark got for the components of the examined competence is combined with the corresponding component of all competences formed in the frames of the given academic course.*

General assessment of maturity level of all competences is made by aggregation of marks got by the student for each component of the formed competences taking into account the results of continuous assessment and progress check in the form of integral mark according to the four-point scale. All control results are put into the assessment sheet by the teacher according to the results of midterm attestation.

The form of the assessment sheet and requirements for its completion are given in the general part of FET of the educational program.

While making the final assessment of the midterm attestation in the form of test standard criteria given in the general part of FET of the educational program are used.